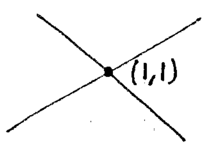
*LINEAR ALGEBRA :.

Analysis

$x+2y=3$
$2x+3y=5$

so, $x=1, y=1$

Intersecting Line



(x' and y')

$x+2y=3$
$2x+4y=6$

let $y=K$

$x=3-2K$

Infinite no. of solutions

COINCIDENT LINE

$x+2y=3$
$x+2y=5$

NO solution

(PARALLEL LINES)

*Any 1st degree 2 dimensional equation in x+y represents a line in the XY PLANE. (LINEAR SYSTEM OF EQUATION IN 2 VARIABLES)

Note :.

*The study of LINEAR SYSTEM OF EQUATIONS is called LINEAR ALGEBRA.

$x+2y=3$
$2x+3y=5$

on solving the equation

$x=1; y=1$

(UNIQUE SOLUTION)

$x+2y=3$
$2x+4y=6$

let $y=K$

$x=3-2K$

(INFINITE NO. OF SOLUTION)

$x+2y=3$
$x+2y=5$

(NO SOLUTION)

*To study about the Linear system of Equations, we require the concept "RANK OF MATRIX". Hence we study about MATRICES in the concept LINEAR ALGEBRA.

*MATRIX :.

*Arrangement of Elements or numbers in Rows and Columns such that each row will have same no. of element and each column will have same no. of element is called a MATRIX.

*Operation on Matrices:

1) Addition
2) Substraction
3) Multiplication $\{A_{m\times l} \times B_{l\times n} = C_{m\times n}\}$
4) TRACE OF SQUARE MATRIX:

*The sum of the PRINCIPAL DIAGONAL ELEMENTS OF A SQUARE MATRIX is called TRACE.

5) SYMMETRIC MATRIX:

When $\boxed{A^T = A}$ the matrix A is ~~skew~~ symmetric

$$\begin{bmatrix} 1 & 5 & -1 \\ 5 & 2 & 9 \\ -1 & 9 & 3 \end{bmatrix}$$

6) SKEW SYMMETRIC MATRIX:

When $\boxed{A^T = -A}$. then Matrix A is SKEW SYMMETRIC.

COMPULSORY CONDITION (diagonal elements should be zero)

$$\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 9 \\ 5 & -9 & 0 \end{bmatrix}$$

*DETERMINANT OF SQUARE MATRIX:

*For a 1x1 MATRIX, the no. ~~itself~~ itself is the Determinant

*For a 2x2 MATRIX of the form:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the determinant is given by. $(ad-bc)$

*MINOR OF AN ELEMENT:

let

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then Minor of $a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (a_{22}a_{33} - a_{32}a_{23})$

Minor of $a_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = (a_{12}a_{33} - a_{32}a_{13})$

* COFACTOR of an Element :-

* Minor of a_{ij} is M_{ij}; then cofactor of a_{ij} is

$$\boxed{\text{cofactor of } a_{ij} = (-1)^{i+j} \cdot M_{ij}}$$

* The Determinant of square matrix is defined as "The sum of product of elements of any row or any column with the corresponding cofactors"

* Analysis :-

let

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

* we have to find the determinant of given 4x4 matrix. for this choose any row or column having the max^m no. of zeroes.

using 2nd column weget:-

$$2(-1)^{3+2} \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 0 \end{vmatrix}$$

$$-2\{1(0+2) - 2(0+1) + 1(2-1)\}$$

$$= -2$$

using 4th column weget

$$1.(-1)^{4+1} \begin{vmatrix} 0 & 2 & 1 \\ 0 & 1 & -1 \\ 2 & 3 & 1 \end{vmatrix} + 2(-1)^{4+3} \begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$-1\{-2(3)+1(-2)\} - 2\{1(+2)+1(2)\}$$

$$+6-8$$

$$=-2$$

$$= -2$$

Note :-

* A matrix is said to be NON ~~SON~~ SINGULAR when

$$\boxed{DET(A) \neq 0}$$

and is said to be SINGULAR when

$$\boxed{DET(A) = 0}$$

** $\boxed{Det(AB) = (Det\ A)(Det\ B)}$

** Det (A+B) is not necessarily (Det A) + (Det B)

** If any two rows are same or constant multiples (columns) then Determinant of that Matrix is zero.

** If ~~same~~ SUM of the elements in every row or every column is zero then the determinant of such matrix is zero.

for eg.

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & -2 \\ 1 & 1 & -2 \end{bmatrix}$$ sum of Rows zero.
(sum of each row is zero).

*ADJOINT OF SQUARE MATRIX :-

*It is the Transpose of cofactor Matrix ie

if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then the cofactor of $a_{ij} = A_{ij}$

then $\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Note :-

* when ** $\boxed{A(\text{adj } A) = (\det A) I}$ $I \rightarrow$ Identity matrix.

* ** $\boxed{\det(\text{adj } A) = (\det A)^{n-1}}$; n= order of matrix

*) ** $\boxed{\text{Adj}(\text{adj } A) = (\det A)^{n-2} A}$

*INVERSE of SQUARE MATRIX :-

* A matrix B is said to be Inverse of a Non singular matrix A if ** $\boxed{AB = BA = I}$

*To find A^{-1} we have

** $\boxed{A^{-1} = \frac{\text{Adj } A}{\text{Det } A}}$

*for Matrix A ; $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

** $\boxed{A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \; ; \; \boxed{ad-bc \neq 0}}$

** * $\boxed{\det(A^{-1}) = \frac{1}{(\det A)}}$

*ELEMENTARY TRANSFORMATION ON A MATRIX:

*There are only 3 elementary transformations; they are:

1) Interchanging of any two rows ($R_1 \leftrightarrow R_2$)

2) Multiplication of a row by a constant ($R_2 \to 3R_2$)

3) Addition of 1 ROW to the corresponding elements of some other row ($R_2 \to R_2 + R_1$).

Note:

* $R_2 \to R_2 + 3$
* $R_2 \to R_2 \times R_1$

} Not elementary xmation.

*Inverse of Matrix (using elementary xmation)

*GAUSS JORDAN METHOD:

Q1) Find the Inverse of

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Use this element to make all the elements below/above this as zero.

Soln:

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$(R_2 \to R_2 - R_1)$; $[R_3 \to (R_3 - R_1)]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$(R_1 \to R_1 - 3R_2)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 4 & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

Hence,

$$\boxed{A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}$$

(Q2) Find the inverse of $A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Soln:. By Gauss Jordan method:.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$(R_1 \to R_1 - 3R_4)$; $(R_2 \to R_2 + 2R_4)$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

So, $\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$

*MINOR OF A MATRIX:.

* For finding the No. of minors of given order choose no. of rows or columns from given no. of Rows or Columns.

let

$$A = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_3 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 \end{bmatrix}_{4\times5}$$

choosing any Row. choosing any columns.

Note:.

(4x4) ✓No. of minors of order 4 is 5. $({}^4C_4 \times {}^5C_4 = 5)$

(3x3) ✓No. of minors of order 3 is ${}^4C_3 \times {}^5C_3 = 4 \times 10 = 40$ (choose any 3 rows or columns)

(2x2) ✓No. of minors of order 2 is ${}^4C_2 \times {}^5C_2 = 6 \times 10 = 60$ (choose any 2 rows or colums).

(1x1) ✓No. of minors of order 1 is $4 \times 5 = 20$.

*In general, for matrix $A_{m \times n}$:.

i) ~~These~~ The no. of minors of order 'r' that can be generated is $({}^nC_r \times {}^mC_r)$.

ii) The order of greatest minor that can be obtained for this matrix is min (m,n).

$A_{5\times2} \Rightarrow A_{2\times2} \to$ greatest minor ∄ No $(A_{3\times3})$.

$A_{3\times7} \to A_{3\times3} \to$ greatest minor ∄ No $(A_{4\times4})$.

RANK OF A MATRIX :.

*Exists for both Square as well as Rectangular matrix.

*A no. "r" is said to be the "RANK OF A MATRIX A" if :.

i) there exist a minor of order "r" of A which is not zero.

ii) all minors of order more than "r" of A must be zero.

for eg :.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 10 \end{bmatrix}$$

*All red dotted minors A > have det = 0.

*Green dotted minor donot have det = 0.

then $\det A = 0$

and $\det \begin{bmatrix} 4 & 6 \\ 6 & 10 \end{bmatrix} \neq 0 \Rightarrow 40-36 = 4$

*Note :. For given 3x3 matrix, the Minor of 3rd order is the given matrix itself. Also the det. of given minor is zero. Hence, also no. other minor of order 4x4 is available. Hence the matrix A cannot have $\rho(A)=3$. Hence we need to search for 2x2 minor and check for availability of such minor whose det $\neq 0$.

Hence, there exist a minor of order 2x2 whose det is not zero. Hence

$$\boxed{\begin{array}{c} \text{Rank} = 2 \\ \rho(A) = 2 \end{array}}$$

← Rank of Matrix can also be defined as the order of Largest non zero minor of the matrix (Here 2x2 minor).

Note :.

*To find the Rank of the matrix we can use ELEMENTARY XMATIONS.

*By converting, the given matrix into its "ECHELON FORM"; "the no. of NON ZERO ROWS in the "ECHELON FORM IN THE MATRIX" represents the rank of the matrix."

Note :. Calculation of Rank through Minor calculation is very time taking. Hence we use Rank calculation through "ECHELON FORM."

*ECHELON FORM :.

*By applying Elementary Transformations we can convert a given matrix into a form in which :.

i) All zero Rows must be present below Non zero Rows.

ii) In the Non zero Rows; the no. of zeroes before the 1st non zero no. to the next row must increase.

*Such a form is called "ECHELON FORM OF GIVEN MATRIX".

Q3) Find the Rank of :.

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

* Note :. Going through MINOR calculations to obtain RANK OF MATRIX is time taking. Hence ECHELON FORM FORMATION is used to calculate the Rank of A MATRIX

$\boxed{\rho(A) = \text{RANK of MATRIX } A}$

$$A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \to 2R_2 + R_1$

$R_3 \to 2R_3 + R_1$

* No Zeroes before (-2)
* 1 Zero before 3. Hence no. of Zero increased from going from 1st row to 2nd row.
* 1 Zero before (-1) hence no increase in no. of zero from 2nd to 3rd Row. Hence not in ECHELON FORM.

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2$

$R_4 \to 3R_4 - R_2$

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

All Zero Row present below Non zero Row.

$\boxed{\rho(A) = 2}$

Note : (Assumption)

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

matrix in Echelon form only.

$\boxed{\rho(A) = 3}$

Q4) Find the Rank of

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

Soln:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$R_4 \to 2R_4 - 5R_1$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -3 & -6 & -9 & -12 \end{bmatrix}$$

$R_3 \to R_3 - R_2$.

$R_4 \to R_4 - 3R_2$.

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\rho(A) = 2}$