01/09/2024

NUMBER SYSTEM

FACTORS / DIVISOR

Factors are the set of no's that will divide a given number completely.

Example :-

① $72 = 2^3 \times 3^2 = 4 \times 3 = 12$

2^0: $3^0 = 1$, $3^1 = 3$, $3^2 = 9$

2^1: $3^0 = 2$, $3^1 = 6$, $3^2 = 18$

2^2: $3^0 = 4$, $3^1 = 12$, $3^2 = 36$

2^3: $3^0 = 8$, $3^1 = 24$, $3^2 = 72$

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

(II) $120 = 2^3 \times 3^1 \times 5^1 = 4 \times 2 \times 2 = 16$ - TF

TF ↓ Total Factor

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120.

✱ $N = a^p \times b^q \times c^r \ldots\ldots$

Total factor = $(p+1)(q+1)(r+1)$

where,

a, b, c are distinct Prime No's.

p, q, r are Natural Numbers.

(III) $10{,}800 = 2^4 \times 5^2 \times 3^2 = 5 \times 3 \times 4 = 60$ - TF

2nd Mtd :- 10800

108×100

$(12 \times 9) \times (5^2 \times 2^2)$

$(2^2 \times 3^1)(3^2) \times (5^2 \times 2^2)$

$= 2^4 \times 3^3 \times 5^2 = 5 \times 4 \times 3 = 60$

Odd Factor = $3 \times 4 = 12$

Even Factor = $(60 - 12) = 48$

(IV) $9000 = 2^3 \times 3^2 \times 5^3 = 4 \times 3 \times 4 = 48$ - TF

Odd factors = $3 \times 4 = 12$

Even factors = $(48 - 12) = 36$

Q1:- How many factors of number 72 are multiple of 6.

Q2: How many factors of number 120 are multiple of 30.

Q3:- How many factors of number 9000 are multiple of 30.

① $72 = 2^3 \times 3^2$

$72 = (2 \times 3)(2^2 \times 3)$

T.F = $3 \times 2 = 6$

② $120 = 2^3 \times 3^1 \times 5^1$

$= (2^2 \times 3^1)(2^1 \times 5^1)$

T.F = $2 \times 2 = 4$

③ $9000 = 2^3 \times 3^2 \times 5^3$

$= (2^1 \times 3^1 \times 5^1)(2^2 \times 3^1 \times 5^2)$

T.F = $3 \times 2 \times 3 = 18$

Prime and Composite Factors

1 is neither prime nor composite.

Total factors = Prime factors + composite factors + 1

Example : $72 = 2^3 \times 3^2$

PF = 2 for (2, 3)

CF = ?

NPNC = 1

TF = 12

$12 = 2 + CF + 1$

CF = 9

NPNC = Neither prime nor composite.

$120 = 2^3 \times 3^1 \times 5^1$

PF = (2,3,5) = 3

CF = ?

NPNC = 1

16 = 3 + CF + 1

∴ CF = 12

$10800 = 2^4 \times 3^3 \times 5^2$

PF = 3

CF = ? NPNC = 1

60 = 3 + CF + 1

∴ CF = 56

9000 = 48 = 3 + CF + 1

CF = 44.

Not Important

$N = a^p \times b^q \ldots\ldots$

Sum of all factors $= \dfrac{(a^{p+1}-1)}{(a-1)} \times \dfrac{(b^{q+1}-1)}{(b-1)} \ldots$

Example :- $2^3 \times 3^2 = 72$

$N = \left(\dfrac{2^4-1}{2-1}\right) \times \left(\dfrac{3^3-1}{3-1}\right) = 15 \times 13 = 195$

Product of all N $= [N]^{\frac{TF}{2}}$

Example :-

① $[72]^{\frac{12}{2}} = 72^6$

② $[120]^{\frac{16}{2}} = [120]^8$

Ex:- $36 = 2^2 \times 3^2 = 3 \times 3 = 9 = TF$

$36^4 \times 6 = 36^4 \times 36^{1/2} = 36^{4.5} = (36)^{1/2} = N^{\left[\frac{TF}{2}\right]}$

1, 2, 3, 4, 6, 9, 12, 18, 36

Base System

$10^1\ 10^0$

$a \cdot b = 10a + b$

$10^2 . 10^1 . 10^0$

$abc = 100a + 10b + c$

Th. Hun Tens Units

3476

$10^3\ 10^2\ 10^1\ 10^0$

$(25)_{10} = (11001)_2$

2	25	Rem.
2	12	1
2	6	0
2	3	0
	1	1

$(25)_{10} = (121)_4$

4	25	Rem.
4	6	1
	1	2

$(25)_{10} = (\overset{2^4}{1}\overset{2^3}{1}\overset{2^2}{0}\overset{2^1}{0}\overset{2^0}{1})_2$

$= (16 + 8 + 0 + 0 + 1)_2$

$= (25)_{10}$

$(25)_{10} = (\overset{4^2}{1}\overset{4^1}{2}\overset{4^0}{1})_4$

$= (16 + 8 + 1)_4 = (25)_{10}$

Q2 | WB 137 + 276 = 435

731 + 672 = ?

$\left[\begin{array}{r} 137 \\ +276 \\ \hline 435 \end{array}\right]_{b=8}$ $\left[\begin{array}{r} 731 \\ +672 \\ \hline 1623 \end{array}\right]_{b=8}$ $\left[\begin{array}{r} \overset{6}{7}\,\overset{2}{3}\,\overset{9}{1} \\ -672 \\ \hline 037 \end{array}\right]_{b=8}$

7 + 6 = b + 5 ⇒ b = 8.

Q:- If $\left[\begin{array}{r} 4226 \\ -2442 \\ \hline 10001 \end{array}\right]_{b=7}$ then $\left[\begin{array}{r} 23\overset{7}{4}2 \\ -1656 \\ \hline 0353 \end{array}\right]_{b=7}$

6 + 2 = b + 1

b = 7

$(7526)_8 - (y)_8 = (4364)_8$

$a - y = b$

$y = a - b$

$$\left[\begin{array}{r} 7\,5\,2\,6 \\ -4\,3\,6\,4 \\ \hline 3\,1\,4\,2 \end{array}\right] b=8$$

Example :

$$\begin{array}{r} 44 \\ \times 11 \\ \hline 1034 \end{array}$$

$b'b^0 \; b'b^0 \quad b^3 b^2 b' b^0$

$44 \times 11 = 1034$

$(4b+4) \times (b+1) = b^3 + 3b + 4$

$4(b+1)^2 = b^3 + 3b + 4$

$\underline{\underline{b=5}}$

$4(6)^2 = 125 + 15 + 4$

$144 = 144$

$$\begin{array}{r} 44 \\ 11 \\ \hline 44 \\ 44\times \\ \hline 1034 \end{array}$$

$\underline{\underline{b=5}}$

CYCLICITY

Power divided by 4 (4 operations)

$4n+1$	$\xrightarrow{\text{Rem1}}$	2	3	7	8
$4n+2$	$\xrightarrow{\text{Rem2}}$	4	9	9	4
$4n+3$	$\xrightarrow{\text{Rem3}}$	8	7	3	2
$4n$	$\xrightarrow{\text{Rem0}}$	6	1	1	6

Operations - 2

4 ← odd power 9 ← odd power

6 ← Even power 1 ← even

Operation-1 Numbers → 0, 1, 5, 6

Example :-

(i) $(732)^{2(\frac{27}{4})} = u = 8$ (Rem.3).

(ii) $(453)^{2(\frac{22}{4})} = u = 9$ (Rem 2)

(iii) $74^{91} = u = 4$

(iv) $79^{91} = u = 9$

(v) $74^{92} = u = 6$

(vi) $79^{92} = u = 1$

(vii) $(76)^{937} \times (34)^{71} + (273)^{993}$

$= 6 \times 4 + 3 = 7$

Q97 | Pg103 | WB

$211^{870} + 146^{127} \times 3^{424} = 1 + 6 \times 1 = 7$

Q103 | Pg107 | WB

$(2171)^7 + (2172)^9 + (2173)^{11} + (2174)^{13}$

$= 1 + 2 + 7 + 4 = 14$

Q148 | WB

$265917\,49^{110016} = 1$

Q4 | WB

$[3^{999} \times 7^{1000}] = 7 \times 1 = 7$

Mtd-2

$(3 \times 7)^{999} \times 7^1 = (21)^{999} \times 7 = 1 \times 7 = 7$

Q:- $(4739)^{2373} \times (22\underset{\text{Even}}{8})^{4532} \times (7357)^{9913} \times (325)^{719} \times (293)^{3213}$

multiple of 5 → $= 0$

$= 9 \times 6 \times 7 \times 5 \times 3$

$= 567\underline{0}$

Even no. & multiple of 5 will result in zero.

Ex:- $1!+2!+3!+4!+5!+\cdots+99! \div 4$ -1

$5! = 5\times4\times3\times2\times1 = 120$

$6! = 6\times5! = 720$

$7! = 7\times6\times5! = 5040$

$1+2+6+24+0+\cdots$

$= 33.$

Q:- No. of zeroes = ? (means power of 10)

$100! = 1\times2\times3\times4\times5\times\cdots\times99\times100$

$\frac{100}{5} = 20\ [5,10,15,20,\cdots,100] \simeq 5^1$

$\frac{20}{5} = 4\ [25,50,75,100] \simeq 5^2$

$\underline{24}$

Q:- $100! = 1\times2\times3\times4\times5\times6\times\cdots\times9\times\cdots\times99\times100.$

$3^n = ?$

$\frac{100}{3} = 33\ [3,6,9,12,\cdots,99] \simeq 3^1$

$\frac{33}{3} = 11\ [9,18,27,\cdots,99] \simeq 3^2$

$\frac{11}{3} = 3\ [27,54,81] \simeq 3^3$

$\frac{3}{3} = 1\ [81] \simeq 3^4$

+

$\underline{48}$

$3^{48} \rightarrow n=48$

Q:- $100! = 7^n = 7^{16}$

$\frac{100}{7} = 14\ [7,14,21,\cdots,98] \simeq 7^1$

$\frac{14}{7} = 2\ [49,98] \simeq 7^2$

+

$\underline{16.}$

Q:- $100! = 15^n$

$(3\times5)^n$

$100! = 1\times2\times3\times4\times5\times\cdots\times100$

$100! = 3^{48}\times5^{24}$

$= (3\times5)^{24}\times3^{24}$

$= (15)^{24}\times\cancel{3^{24}} \rightarrow$ 24 Ans

Q:- If first 9 multiples of 5 are multiplied.

$p = (5\times10\times15\times20\times\cdots\times45)$

no. of zeroes in p.

(a) 4 (b) 9 (c) 10 (d) NOTA

Q2- $\frac{100!}{(77)^n}$, $\frac{100!}{(85)^n}$, $\frac{100!}{(91)^n}$, $\frac{100!}{(65)^n}$

① $5\times10\times15\times20\times30\times\cdots\times40\times\cdots\times45$

$(5\times1)\ (5\times2)\ (5\times3)\cdots\cdots\times(5\times9)$

$1+2+1+3 = 7.$

Mtd: 2 $5^9\times9!$

$5^{10}\times27$

$10^7\times5^3.$

② $100! = 2^{97}\times3^{48}\times5^{24}\times7^{16}\times11^9\times13^7\times17^5\times19^5$

$\frac{100!}{(77)^n} = \frac{100!}{(7\times11)^n} = 9$

$\frac{100!}{(91)^n} = \frac{100!}{(13\times7)} = 7$

$\frac{100!}{(85)^n} = \frac{100!}{(17\times5)^n} = 5$

$\frac{100!}{(65)^n} = \frac{100!}{13\times5} = 7$

100!

$\frac{100}{2} = 50$

25

12

6

3

1

$\underline{97}$

Series

02/09/2024

Q:- $5 \times 10 \times 15 \times \cdots\cdots \times 120$

(5×1) (5×2) (5×24)

$24! =$ No. of 2's

$\frac{24}{2} = 12 \quad \frac{12}{2} = 6 \quad \frac{6}{2} = 3 \quad \frac{3}{2} = 1$

$12 + 6 + 3 + 1 = 22$

Q:- No. of trailing zeros

(a) $1^1 \times 2^2 \times 3^3 \times 5^5 - 10^{10}\ 15^{15} - 25^{25} \times 100^{100}$ $\quad 5050$

$[5 + 10 + 15 + \cdots 25 \cdot + 100]$

$5(1 + 2 + 3 + \cdots + 20)$

$\left\{ \Sigma n = \frac{n(n+1)}{2} \right.$

$5 \times \left[\frac{20 \times 21}{2}\right] = 1050 + 25 + 50 + 75 + 100$

$= 1300.$

$25^{25} = (5^2)^{25} = 5^{50}$

$(50)^{50} = (5^2 \times 2)^{50} = 5^{100}$

$(75)^{75} = (5^2 \times 3)^{75} = 5^{150}$

$100^{100} = (5^2 \times 2^2)^{100} = 5^{200}$

(b) $1! \times 2! \times 3! \times 4! \times \cdots\cdots \times 100!$.

Q25, Pg-95, WB 2025

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

$$= \left(\frac{1}{\sqrt{2}+\sqrt{1}}\right)\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right) + \left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) + \cdots$$

$$\left(\frac{1}{\sqrt{4}+\sqrt{3}}\right)\left(\frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}\right) + \cdots + \left(\frac{1}{\sqrt{81}+\sqrt{80}}\right)\left(\frac{\sqrt{81}-\sqrt{80}}{\sqrt{81}-\sqrt{80}}\right)$$

$$= (\cancel{\sqrt{2}} - \sqrt{1}) + (\cancel{\sqrt{3}} - \cancel{\sqrt{2}}) + (\cancel{\sqrt{4}} - \cancel{\sqrt{3}}) + \cdots + (\sqrt{81} - \cancel{\sqrt{80}})$$

$= \sqrt{81} - \sqrt{1}$

$= 9 - 1 = 8$

Q124, Pg 107, WB

$a_n = \frac{1}{n} - \frac{1}{n+2} \qquad n > 0$

$$\begin{matrix} a_1 & a_2 & a_3 & \cdots\cdots + a_{48} \end{matrix}$$

$$\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{48}} - \cancel{\frac{1}{50}}\right)\right.$$

$$\left. + \underbrace{\left(\cancel{\frac{1}{49}} - \frac{1}{51}\right)}_{a_{49}} + \underbrace{\left(\cancel{\frac{1}{50}} - \frac{1}{52}\right)}_{a_{50}}\right]$$

$$= \left(1 + \frac{1}{2}\right) - \left(\frac{1}{51} + \frac{1}{52}\right)$$

Q:- $S = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \cdots + \frac{1}{399}$

Then $S = ?$

(A) $\frac{20}{21}$ (B) $\frac{19}{21}$ (C) $\frac{11}{21}$ (D) $\frac{10}{21}$

$$S = \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \cdots + \frac{1}{19\times21}$$

$$S = \frac{1}{2}\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) \cdots + \left(\cancel{\frac{1}{19}} - \frac{1}{21}\right)\right]$$

$$S = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{21}\right] = \frac{1}{2}\left[\frac{20}{21}\right] = \frac{10}{21}$$

Mtd: 2

$$\frac{1}{(2^2 - 1^2)} + \frac{1}{(4^2 - 1^2)} + \frac{1}{(6^2 - 1^2)} + \cdots + \frac{1}{(20^2 - 1^2)}$$

$$= \frac{1}{(1\times3)} + \frac{1}{3\times5} + \frac{1}{5\times7} + \cdots + \frac{1}{(19\times21)}$$

$[a^2 - b^2 = (a-b)(a+b)]$

Q:- $\frac{1}{1\times2} + \frac{1}{2\times3} + \cdots + \frac{1}{80\times81}$

$$= \frac{1}{1}\left[1 - \frac{1}{81}\right] = \frac{80}{81}$$

$\frac{1}{1}$: Subtracting (2-1), (3-2) all 1.

1: 1st term

$\frac{1}{81}$: Last Term

Q:- If Red light flashes every 3 secs. and Green light flashes every 4 secs. at what time both flashes together?

$R \xrightarrow{3\text{secs}} 3, 6, 9, 12, \ldots, 24, \ldots, 36, \ldots$

$G \xrightarrow{4\text{secs}} 4, 8, 12, \ldots, 24, \ldots, 36, \ldots$

$K\ LCM\ [t_{R_1}, t_{G_1}]$

$K\ LCM\ [3, 4] = 12\ K$ secs.

LCM Indicates

→ Things happen for the first time

→ Least value

→ Smallest Natural Number.

In one minute time how many times will they be flashing.

⇒ 1 min = 60 secs.

$\frac{60}{12} = 5$ times $[12, 24, 36, 48, 60]$

Q:- R [3 times → 2 min] → 120 secs.

G [5 times → 3 min] → 180 secs.

$(LCM[t_{R_1}, t_{G_1}]) \times K$

$LCM\ [40, 36] = 360$ secs. $\simeq$ 6 mins

$1\text{hr} = \frac{60 \times 60}{360} = 10$ times.

★ $\boxed{LCM\left[\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right] = \frac{LCM(a, c, e)}{HCF(b, d, f)}}$

→ 2nd Mtd :-

$K\ LCM\left[\overset{t_{R_1}}{\frac{2}{3}}, \overset{t_{G_1}}{\frac{3}{5}}\right] = K\ \frac{6}{1}$ min

$1\text{hr} = \frac{60\text{min}}{6} = 10$ mins.

★ If question was both light started flashing in the beggining then first flash will be at $t = 0$ sec. then Add 1.

★ $\boxed{HCF\left[\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right] = \frac{HCF(a, c, e)}{LCM(b, d, f)}}$

Q:- In a school how many times bell rangs bell sangs at the same time if classes are of duration :-

$LCM\left[\overset{VIII}{24}\quad \overset{IX}{30}\quad \overset{X}{40}\quad \overset{XII}{60}\right]$ min

School runs from 8:00 AM → 2:30 PM

→ 120 mins → 2 hrs.

[10 Am, 12 Noon, 2pm]

Q:- HCF of two No's = 24

Sum of these two No's = 144.

What are the Numbers?

→ $N_1 = 24x$ {x & y should be co-prime no's}

$N_2 = 24y$

$N_1 + N_2 = 144$

$24(x + y) = 144$

$x + y = 6$

$1, 5$ — $N_1 = 24 \times 1 = 24$, $N_2 = 24 \times 5 = 120$] $\frac{HCF}{24}$

~~2, 4~~ — $N_1 = 24 \times 2 = 48$, $N_2 = 24 \times 4 = 96$] $\frac{HCF}{48}$ X

~~3, 3~~ — $N_1 = 24 \times 3 = 72$, $N_2 = 24 \times 3 = 72$] $\frac{HCF}{72}$ X

★ If HCF of No's are same then the difference of numbers will have same HCF.

Q29 | Pg-89 | WB

78, 104, 117, 169

$l = HCF\,[78, 104, 117, 169] = 13$

$n_1 l \quad n_2 l \quad n_3 l \quad n_4 13.$

$6 + 8 + 9 + 13 = 36.$

Q:- Three pieces of cake of weight $4\frac{1}{2}, 6\frac{3}{4}, 7\frac{1}{5}$ are to be divided into parts of equal weight further each part must be as heavy as possible. If one such part is to be given to one guest. How many guests can be served.

$\rightarrow 4\frac{1}{2},\ 6\frac{3}{4},\ 7\frac{1}{5}$

$n_1 w \quad n_2 w \quad n_3 w$

$W = \frac{HCF}{LCM} = \left[\frac{9}{2}, \frac{27}{4}, \frac{36}{5}\right] = \frac{9}{20}$

$10 + 15 + 16 = 41.$

* Use of LCM & HCF is phenomenal

Remainder

$\boxed{N = \text{Remainder Mod (Divisor)}}$

If $x = y$ Mod M then definitely $x - y = 0$ Mod M.

Example:-

(I) $80 = 8$ Mod (9) (4) $80 = -1$ Mod (9)

$72 = 0$ Mod (9) (04) $81 = 0$ Mod (9)

(II) $24 = 3$ Mod (7) (04) $24 = -4$ Mod (7)

$21 = 0$ Mod (7) (04) $28 = 0$ Mod (7)

Rule 1 (applicable for +, ×, −)

$a = b\,M(c)$

$d = e\,M(c)$ [M = Mod]

$g = f\,M(c)$

$a \times d \times g = b \times e \times f$ Mod (c)

$b \times e \times f < c$

Example:

(1) $\frac{\overset{5}{\cancel{1421}} \times \overset{7}{\cancel{1423}} \times \overset{9}{\cancel{1425}}}{\cancel{12}} = 315$

∵ Remainder ≮ Divisor.

∴ Divide again.

$\frac{315}{12} = 3.$

• $a + d + g = b + e + f$ Mod (c)

$= b + e + f < c$

• $a + d - g = b + e - f$ Mod (c)

$= b + e - f < c$

Example—

(2) $\frac{\overset{5}{\cancel{1421}} + \overset{7}{\cancel{1423}} + \overset{9}{\cancel{1425}}}{\cancel{12}} = 21,\ \frac{21}{12} = 9$

(3) $\frac{\overset{5}{\cancel{1421}} + \overset{7}{\cancel{1423}} - \overset{9}{\cancel{1425}}}{\cancel{12}} = 3$

Rule-2

$a = b$ Mod(c)

$a^n = b^n$ Mod (c)

$b^n < c$

n = Natural Numbers

Example :-

① $2^{600} \div 15$ = Remainder = ?

$2^4 = 1 \; M(15)$

$(2^4)^{150} = 1^{150} \; M(15)$

$2^{600} = 1 \; M(15)$

Re = 1

② $\dfrac{10^{10 \to 1} + 10^{100 \to 1} + 10^{1000 \to 1} + 10^{10000 \to 1}}{3} = \dfrac{4}{3}$ Re = ?

$10 = 1 \; M(3)$

$(10)^{10} = (1)^{10} \; M(3)$

$10^{10} = 1 \; M(3)$

$(10)^{100} = (1)^{100} M(3)$

$= 1 \; M(3)$

$(10)^{1000} = (1)^{1000} M(3)$

$= 1 \; M(3)$

$(10)^{10000} = (1)^{10000} M(3)$

$= 1 \; M(3)$

Remainder $= \dfrac{1+1+1+1}{3} = \dfrac{4}{3} = 1$

Q:- What is the remainder ?

$\dfrac{2^{192}}{6}$

ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 4

$\dfrac{2^{192 \to \text{even}}}{6}$

$\Rightarrow 2^{1 = \text{odd}} = 2 \; M(6)$

$2^{2 - \text{even}} = 4 \; M(6)$

$2^{3 - \text{odd}} = 2 \; M(6)$

$2^{4 - \text{even}} = 4 \; M(6)$

$2^{5 - \text{odd}} = 2 \; M(6)$

$2^{6 - \text{even}} = 4 \; M(6)$

$2^{7 = \text{odd}} = 2 \; M(6)$

$2^{8 = \text{even}} = 4 \; M(6)$

$\dfrac{2^{192}}{6} = \dfrac{\cancel{2} \times}{\cancel{2} \times}\left(\dfrac{2^{191}}{3}\right)$

$2 = (-1) \; M(3)$

$2^{191} = (-1)^{191} \; M(3)$

$2^{191} = -1 \; M(3)$

$2^{191} = 2 \; M(3)$

Eg:- $11 = -1$ Mod 3
$11 = +2$ Mod 3

$\dfrac{2^{191}}{3}$ = Re 2. $\times$ 2 = Re. 4.
(2 cancelled out)

Q: $5^{625} \div 7$ Re = ?

$5^3 = 6 \; M(7)$

$5^3 = -1 \; M(7)$

$(5^3)^{208} = (-1)^{208} \; M(7)$

$5^{624} = 1 \; M(7)$

$5^1 = 5 \; M(7)$

$5^{625} = 5 \; M(7)$

Mtd-2

$5^2 = 4 \; M(7)$

$(5^2)^3 = (4)^3 \; M(7)$

$5^6 = 64 \; M(7)$

$5^6 = 1 \; M(7)$

$(5^6)^{104} = (1)^{104} M(7)$

$5^{624} = 1 \; M(7)$ next

Mtd-3

$5 = -2 \; M(7)$

$(5)^3 = (-2)^3 \; M(7)$

$5^3 = -8 \; M(7)$

$5^3 = -1 \; M(7)$ next

PERCENTAGE

Q1. A's salary is 20% more than that of B. By how much percentage is B's salary is less than that of A.

$$\frac{x}{100+x} \rightleftarrows \frac{x}{100-x}$$

↑ 10 ⇄ 9.09 ↓

↑ 20 ⇄ $16.\bar{6}$ ↓

↑ 25 ⇄ 20 ↓

↑ $33.\bar{3}$ ⇄ 25 ↓

↑ 50 ⇄ $33.\bar{3}$ ↓

* {For increment → Red one↑
For decrement → Green one↓}

In this question, Increment

So, Answer = $x = 20$

$$\frac{20}{100+20} = \frac{20}{120} = \frac{1}{6} = 16.\bar{6}\%$$

Q2. A's salary is 20% less than that of B. By how much % is B's salary is more that that of A.

In this question, decrement,

So, $\frac{20}{100-20} = \frac{20}{80} = \frac{1}{4} = 25\%$

Method : 2

① B = 100 A = 120 20% ↑

$$\frac{-20}{120} = -\frac{1}{6} = \downarrow 16.\bar{6}\%$$

② B = 100 A = 80 20% ↓

$$\frac{+20}{80} = \frac{1}{4} = 25\%$$

* $R = a \times b$

$a \to x\%$, $b \to y\%$

$$\Delta R\% = x + y + \frac{xy}{100}$$

$$\Delta R\% = x+y+z+\frac{xy+yz+zx}{100}+\frac{3xyz}{100^2}$$

$x+\frac{10}{100}x$	$x-\frac{10}{100}x$
$x[1+.1]$	$x[1-.1]$.
$1.10x$	$.90x$
$1.20x$	$.80x$
$1.23x$	$.77x$

$A = l \times b$.

$D = s \times t$

$R = p \times N$

Example:-

① $l = 20\uparrow^{+}$, $b = 10\uparrow^{+}$

$A = 1\ lb$

$A' = 1.2l \times 1.1b$

$A' = 1.32\ lb. = 32\%\uparrow$

2nd Mtd :- $20+10+\frac{20\times10}{100}$

$= 32\%$

3rd Mtd $1.2\times1.1 = 1.32 = 32\%\uparrow$

② $l = 20\uparrow^{+}$ $b = 10\downarrow^{-}$

$A = 1\ lb$

$A' = 1.2l \times 0.9b$

$A' = 1.08\ lb = 8\%\uparrow$

2nd Mtd:- $20+(-10)+\frac{20\times(-10)}{100}$

$= 8\%\uparrow$

3rd Mtd:- $1.2 \times 0.9 = 1.08 = 8\%\uparrow$